Dispersion Characteristics of Twisted Rectangular Waveguides

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Abstract—**Based on the expressions for the dominant hybrid-mode fields in twisted rectangular waveguides, dispersion formulas with two perturbational factors have been derived theoretically. These factors correspond to the two effects of the twist on the dominant wave propagation, respectively. The one expresses a shift in the cutoff frequency while the other expresses the effect of elongation in the transmission path.**

A set of 20-cm-long waveguides twisted uniformly by various multiples of 90° has been manufactured by the method of electroforming. The resonant frequencies of the respective waveguides have been measured as a transmission cavity in the 10-GHz band to obtain dispersion relations. Experimental results are found to be in good agreement with the theoretically derived formulas. The results of Lewin's theory are also compared with the present ones.

I. Introduction

A THEORETICAL analysis of propagation constants in twisted rectangular waveguides has already been introduced by Lewin [1]. In the course of Lewin's work, electromagnetic fields in the waveguide have been assumed to be of the TE-mode type whose electric field lies entirely on the plane perpendicular to the guide axis. On the other hand, the present authors have investigated thoroughly this boundary value problem, and have derived the hybrid-mode fields that exactly satisfy the boundary conditions on the guide walls in helicoidal shape [2].

The purpose of this study is to provide some basic data for designing an optimum waveguide twist which has an allowable reflection coefficient with minimum length for the required bandwidth.

The present paper is concerned with the argument on the hybrid-mode fields and it starts from the relations of stored energies in this mode. Formulas for the dispersion relations of the twisted waveguide are given in the same form as those for a straight one, except for two perturbational factors. The effects of the twist on the formulas can be interpreted intuitively through these factors.

The results of experimental investigations are also presented in this paper. A set of waveguides with various twist angles is employed for the experiments to prove the theoretical formulas. By paying special attention to the cross-sectional geometries of tested waveguides, an alternative method is proposed for evaluating the cross-sectional dimensions. Results of the resonant frequency measurements in the 10-GHz band are shown to be in good agreement with the predictions by the present theory.

Detailed comparisons of Lewin's theory with the present ones are made, and the discrepancies between these two are clarified.

II. Theoretical Formulations

A. Determination of Phase Constants

In a preceding paper [2], the present authors derived the expressions of dominant hybrid-mode fields in uniformly twisted rectangular waveguides with perfectly conducting walls. For a propagating mode in such structures, it is well known that the time-average electric and magnetic energies associated with a specific mode are equal [3].

Applying the energy relation to our problem, we obtain

$$\iint_S \operatorname{rot} E \cdot \operatorname{rot} E^* \, dS = k^2 \iint_S E \cdot E^* \, dS \tag{1}$$

where $S = ab$ denotes the cross section of the twisted waveguide, a the width, b the height, and k the free-space wavenumber. The covariant components (E_ξ, E_η, E_ζ) of the electric field vector E in twisted coordinates (X, Y, Z) have been found [2] to be

$$\begin{aligned} E_\xi &= \alpha \Phi_\xi^{(1)} \exp(-j\beta_T Z) \\ E_\eta &= \left(\Phi_\eta^{(0)} + \alpha \Phi_\eta^{(1)}\right) \exp(-j\beta_T Z) \\ E_\zeta &= \alpha \Phi_\zeta^{(1)} \exp(-j\beta_T Z) \end{aligned} \tag{2}$$

in which the twist constant α (rad/m) is assumed to be small. Since the twisted coordinate system, as well as the procedure for deriving the hybrid-mode fields, have already been described in detail in the previous paper [2], only the expressions of the scalar functions $\Phi_\eta^{(0)}$, $\Phi_\xi^{(1)}$, $\Phi_\eta^{(1)}$, and $\Phi_\zeta^{(1)}$ will be given in the Appendix.

The perturbed phase constant β_T of the hybrid-mode may be written as

$$k^2 - \beta_T^2 = \frac{\pi^2}{a^2}\left(1 + A_2\alpha^2 + \cdots\right) \tag{3}$$

where π/a is the unperturbed eigenvalue of the TE_{10}-mode, and A_2 is a constant to be determined. Here we assume that this expression is valid in the second-order approximation with respect to α.

Substitution of (2) and (3) into (1) gives (see Appendix)

$$A_2 = \frac{a^2}{4\pi^2}\left[A_2'(\phi) + \left(\frac{ka}{\pi}\right)^2 A_2''(\phi)\right] \tag{4}$$

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in which

$$A_2'(\phi)=\frac{8}{3}\phi^2-6-\pi^2+\frac{128}{\pi^2}\cdot\frac{\tan\phi}{\phi}+\frac{256}{\pi^2\phi}\cdot\sum_{m=2,4,\cdots}^{\infty}\frac{(m^2+1)^2\tanh\left(\phi\sqrt{m^2-1}\right)}{\left(\sqrt{m^2-1}\right)^9} \quad (5a)$$

$$A_2''(\phi)=-\frac{4}{3}\phi^2+6+\pi^2-\frac{128}{\pi^2}\cdot\frac{\tan\phi}{\phi}-\frac{256}{\pi^2\phi}\cdot\sum_{m=2,4,\cdots}^{\infty}\frac{(3m^2+1)\tanh\left(\phi\sqrt{m^2-1}\right)}{\left(\sqrt{m^2-1}\right)^9} \quad (5b)$$

where $\phi=\pi b/2a$, and b/a is referred to as the waveguide aspect ratio.

B. Dispersion Formulas

Upon introducing (4) into (3), we obtain various dispersion formulas for twisted rectangular waveguides as follows:

Elliptic relation;
$$\frac{1}{r_1^2}\left(\frac{\lambda}{2a}\right)^2+\frac{1}{r_2^2}\left(\frac{\lambda}{\lambda_T}\right)^2=1 \quad (6a)$$

Guide wavelength;
$$\lambda_T=\frac{\lambda}{r_2\sqrt{1-\left(\frac{\lambda}{2r_1a}\right)^2}} \quad (6b)$$

Phase constant;
$$\beta_T=r_2\sqrt{\left(\frac{2\pi f}{c}\right)^2-\left(\frac{\pi}{r_1a}\right)^2} \quad (6c)$$

Cutoff frequency;
$$f_{cT}=\frac{c}{2r_1a}=\frac{f_c}{r_1} \quad (6d)$$

where λ is the free-space wavelength, c is the velocity of light, and f_c is the cutoff frequency in the straight waveguide with the same cross section. Further, r_1 and r_2 in (6a)–(6d) are given as follows:

$$r_1=\sqrt{\frac{1-A_2''(\phi)\bar{\alpha}^2}{1+A_2'(\phi)\bar{\alpha}^2}} \quad (7a)$$

$$r_2=\sqrt{1-A_2''(\phi)\bar{\alpha}^2} \quad (7b)$$

where $\bar{\alpha}=\alpha a/2\pi$ denotes the degree of twist as a dimensionless quantity.

The newly defined factors r_1 and r_2 play a very important role in the dispersion relations. If the degree of twist $\bar{\alpha}$ is equal to zero, both of them become unity, respectively, and the formulas (6a)–(6d) are then reduced to those for well-known straight waveguides. The effects of the twist on the values of r_1 and r_2 are presented graphically in Fig. 1.

In order to give a better interpretation to r_1 and r_2, we rewrite the formula (6c) as

$$\beta_TZ=r_2Z\sqrt{\left(\frac{2\pi f}{c}\right)^2-\left(\frac{\pi}{r_1a}\right)^2}. \quad (8)$$

Hence, we can recognize the role of r_1 and r_2 as multiplying factors to a and Z, respectively. Thereupon, the concepts of equivalent guide width r_1a and equivalent path length r_2Z can be introduced. Since r_1 decreases with increasing $\bar{\alpha}$, the equivalent guide width is considered to be diminished by twisting. On the other hand, r_2 increases by twisting and it causes an elongation of the equivalent length of the transmission path. This means that the wave propagates along a helical path slightly deviated from the central axis of the twisted waveguide.

The effects of the twist on the dispersion formulas (6a)–(6c) are illustrated in Fig. 2.

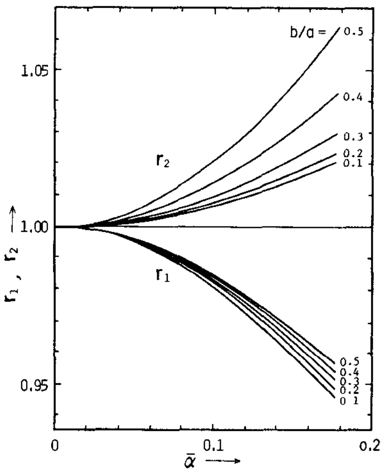

Fig. 1. Plot of the factors r_1 and r_2 versus the degree of twist $\bar{\alpha}=\alpha a/2\pi$ with the waveguide aspect ratio b/a as a parameter.

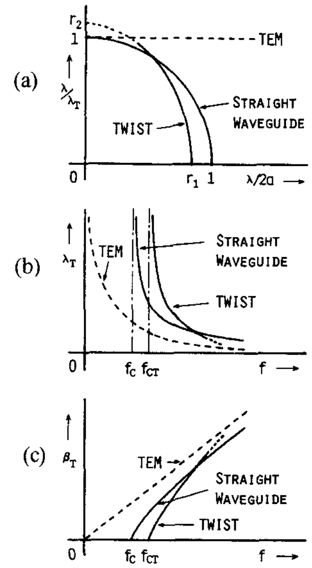

Fig. 2. The effects of the twist on various expressions for the dispersion characteristics. (a) λ/λ_T versus $\lambda/2a$. (b) λ_T versus f. (c) β_T versus f. Dispersion relations for a TEM wave are also shown for comparison.

III. Experimental Investigation

A. Twisted Waveguides Under Test

The 20-cm-long specially made waveguides are shown in Fig. 3, which are of standard cross section compatible with WRJ-10 (22.9 mm×10.2 mm). The respective waveguides are twisted uniformly with angles from 0° (straight) to

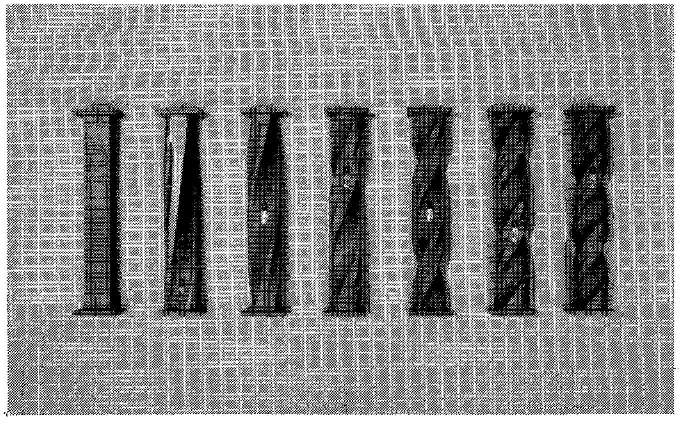

Fig. 3. A set of twisted waveguides under test.

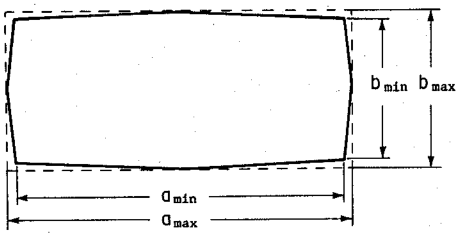

Fig. 4. An exaggerated illustration of waveguide cross section.

TABLE I
DIMENSIONS OF SAMPLE WAVEGUIDES

Twist angle	a (mm)		b (mm)		ℓ (mm)	
	max.	min.	max.	min.	average	s.d.
0°	22.90	22.89	10.20	10.19	199.650	0.006
90°	22.90	22.86	10.21	10.15	200.110	0.007
180°	22.90	22.82	10.20	9.96	200.099	0.011
270°	22.90	22.76	10.20	9.73	200.136	0.013
360°	22.89	22.63	10.21	9.38	200.062	0.007
450°	22.89	22.51	10.22	9.05	200.130	0.005
540°	22.88	22.44	10.21	8.44	200.143	0.005

540° at 90° intervals. These seven samples have been manufactured by the method of electroforming, using aluminum formers as the disposable cores. The formers have been manufactured by using a digital machining system to keep their specified cross-sectional dimensions constant throughout the length of the structures.

Dimensional inspections of the aluminum formers for the respective twisted waveguides were carried out by a three-dimensional measuring machine equipped with data processors. It was found that the larger the degree of twist, the more the deformation in cross-sectional geometries of the respective structures. Fig. 4 shows an exaggerated illustration of the deformed cross section in a typical case. The irregularities in the cross-sectional dimension along axial lengths were found to be within ± 5 μm, which may be considered negligible as compared with the systematic deformation. Dimensions of the sample waveguides are listed in Table I, where the meanings of the maximum and minimum values of a and b are explained in Fig. 4. The thickness of the 7-μm silver plating is ignored there. The

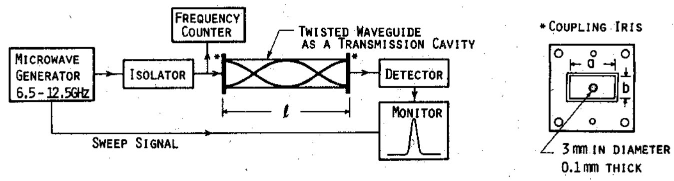

Fig. 5. Arrangement of equipment for the measurement of resonant frequencies by the transmission cavity method.

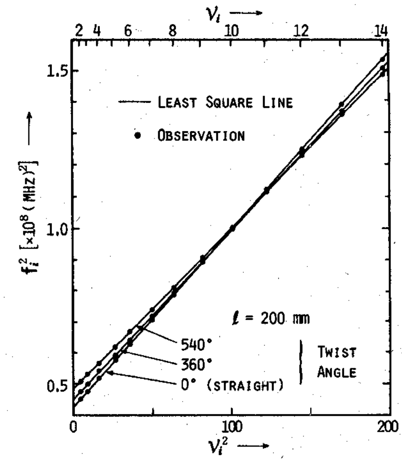

Fig. 6. Plot of f_i^2 versus ν_i^2 and the least-square regression lines, where f_i denotes the observed resonant frequency and ν_i the number of half-guide wavelengths.

respective axial lengths l are shown in terms of the average values and standard deviations for repeated measurements.

B. Resonant Frequency Measurement

Let us now consider the case where a twisted waveguide of axial length l constitutes a resonant cavity. Then, the resonant guide wavelength λ_{Ti} can be expressed as

$$\lambda_{Ti} = \frac{2l}{\nu_i} \tag{9}$$

where ν_i is an integer corresponding to the number of half-guide wavelengths. Substitution of (9) into (6a) or (6b) gives

$$f_i^2 = \left(\frac{c}{2lr_2}\right)^2 \nu_i^2 + \left(\frac{c}{2ar_1}\right)^2 \tag{10}$$

where f_i is a resonant frequency in ν_ith order. Equation (10) gives a linear relationship between f_i^2 and ν_i^2, which is to be corroborated experimentally.

Resonant frequencies of the respective waveguides were then measured by a transmission cavity method [4] as shown in Fig. 5. The equivalent electrical lengths of the coupling irises at both ends were carefully considered, and their effects in the order of 0.1 percent were corrected by using the measured data for a straight waveguide. The resonant frequencies ranging from 6714.5 MHz to 12375.2 MHz corresponding to $\nu_i = 2$ to 14 were observed in the respective waveguides.

Fig. 6 shows the plot of the measured values of f_i^2 versus ν_i^2 together with the least-square regression lines, in which only three typical examples are plotted to illustrate the linear relationship predicted by the theory.

C. Evaluation of Waveguide Cross-Sectional Dimensions

Further investigation on the perturbational factors r_1 and r_2 introduced in the theoretical dispersion formulas will be made here. Calculation of these factors directly from (7a) and (7b) necessitates the value of ϕ, and, hence, the value of waveguide aspect ratio b/a. The cross-sectional dimensions of the sample waveguides, however, have appreciable deviations from the specified values as listed in Table I. Therefore, the ratios b/a have an ambiguity due to the maximum and minimum values of a and b, especially for the waveguides with twist angles of 270° or larger. Thus, instead of simply adopting the measured dimensions, we now present an alternative method for evaluating the cross-sectional dimensions as follows. Considering the results obtained from the resonant frequency measurements, we rewrite the formula (10) in the form

$$f_i^2 = p\nu_i^2 + q \tag{11}$$

where p and q can be determined from the least-square regression lines in Fig. 6. On the other hand, substitution of (7a) and (7b) into (10) and comparison with (11) yield the following simultaneous equations for ϕ and a:

$$p = \left(\frac{c}{2l}\right)^2 \frac{1}{1 - A_2''(\phi)\cdot\left(\frac{\alpha a}{2\pi}\right)^2} \tag{12a}$$

$$q = \left(\frac{c}{2a}\right)^2 \frac{1 + A_2'(\phi)\cdot\left(\frac{\alpha a}{2\pi}\right)^2}{1 - A_2''(\phi)\cdot\left(\frac{\alpha a}{2\pi}\right)^2}. \tag{12b}$$

Elimination of a from these equations gives a transcendental equation as shown as follows:

$$\left[A_2''(\phi) - \left(\frac{c^2}{4l^2p} - 1\right)\cdot A_2'(\phi)\right]\left(\frac{\alpha l}{2\pi}\right)^2 + \frac{q}{p}\left(\frac{c^2}{4l^2p} - 1\right) = 0. \tag{13}$$

Since p and q are already known, the value of ϕ can be determined as a solution of this equation, if we use the measured values of l and α.

When ϕ is evaluated, the values of a, and, hence, b, can then be determined as follows:

$$a = \frac{l}{\sqrt{\frac{q}{p} - A_2'(\phi)\cdot\left(\frac{\alpha l}{2\pi}\right)^2}} \tag{14a}$$

$$b = \frac{2\phi l}{\pi\sqrt{\frac{q}{p} - A_2'(\phi)\cdot\left(\frac{\alpha l}{2\pi}\right)^2}} \tag{14b}$$

in which a can be evaluated even when $\alpha = 0$, but ϕ and b become indeterminable when $\alpha = 0$. Hence, it follows that the values of ϕ and b depend critically on the measured values of p, q, and l for slightly twisted samples. This means, in other words, that the method for evaluating the cross-sectional dimensions proposed here is effective rather for rapidly twisted samples.

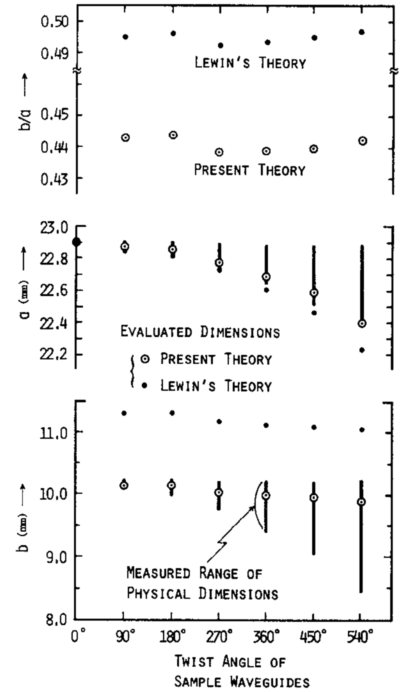

Fig. 7. Evaluated cross-sectional dimensions of the sample waveguides. Ranges of the physical dimensions a and b are also shown for comparison.

The value of a and b evaluated from (14a) and (14b) are to be compared with their physical dimensions in Table I. The results will be discussed in the following section.

IV. Results and Discussions

The expression of the guide wavelength derived by Lewin [1, p. 79, eq. (44)] can be rearranged in comparison with the relation (3) in Section II-A, and the expressions corresponding to (5a) and (5b) are given as follows:

$$A_{2L}'(\phi) = \frac{8}{3}\phi^2 - 4 - \pi^2 + \frac{128}{\pi^2}\cdot\frac{\tan\phi}{\phi} + \frac{256}{\pi^2\phi} \cdot \sum_{m=2,4,\cdots}^{\infty} \frac{(2m^2+1)\tanh\left(\phi\sqrt{m^2-1}\right)}{\left(\sqrt{m^2-1}\right)^9} \tag{15a}$$

$$A_{2L}''(\phi) = -\frac{4}{3}\phi^2 + 6 + \pi^2 - \frac{128}{\pi^2}\cdot\frac{\tan\phi}{\phi} - \frac{256}{\pi^2\phi} \cdot \sum_{m=2,4,\cdots}^{\infty} \frac{(2m^2+1)\tanh\left(\phi\sqrt{m^2-1}\right)}{\left(\sqrt{m^2-1}\right)^9} \tag{15b}$$

where subscript L denotes the Lewin's theory. The results of the present theory are derived from the dominant hybrid-mode fields, while those of the Lewin's theory are from the dominant TE-mode fields. The differences between them are mainly seen in the terms under the sign of summation.

The values of b/a, a, and b obtained, respectively, from (13), (14a), and (14b) are shown in Fig. 7 in comparison with those from Lewin's theory. The ranges of the physical

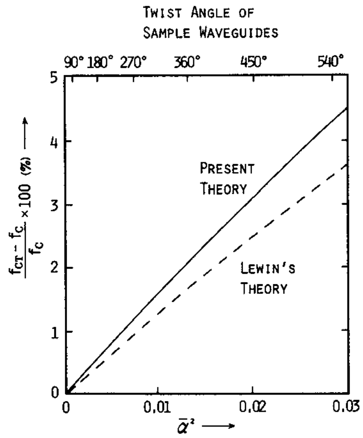

Fig. 8. Plot of the relative increment of the cutoff frequency versus square of the degree of twist.

dimensions for a and b listed in Table I are also indicated by vertical bars. Each value of a and b evaluated from the present theory agrees well with the respective dimensions within the range of variations. But, on the contrary, all of the values from Lewin's theory are out of the ranges in Fig. 7.

By employing the value of b/a and a evaluated from the present theory, the dispersion characteristics of the respective waveguides can be expressed numerically by introducing them into the formulas (6a)–(6d). As an example, a relative increment in the cutoff frequency plotted versus square of the degree of twist is shown in Fig. 8. The solid line indicates the results of the present theory calculated from (6d) together with (5). The dotted line indicates the Lewin's theory calculated from (6d) together with (15) instead of (5). Both results are obtained by introducing the same value of ϕ evaluated from the present theory for the respective samples. Discrepancies between these two are solely caused by the differences between (5) and (15).

V. Conclusions

The dispersion formulas (6a)–(6d) for the twisted rectangular waveguides have been obtained on the basis of the energy relation in the dominant hybrid-mode fields. By examining the two perturbational factors in the formulas, the concepts of equivalent guide width and equivalent path length have been derived. It is found that the effects of the twist are equivalent to the contraction in the guide width and the elongation in the path length. As a result, the cutoff frequency becomes higher with the increasing angle of the twisting. Hence, the guide wavelength increases in the low-frequency region near cutoff. But, in higher frequencies far beyond cutoff, it decreases and becomes shorter than that of the straight waveguide. The effects of the twist are explained more intuitively with the aid of the diagrams as shown in Fig. 2.

By using a set of twisted waveguides, experimental investigations have been carried out to corroborate the theoretical dispersion formulas. Equations (13), (14a), and (14b) have been derived as useful relations for evaluating the cross-sectional dimensions of the twisted waveguides. Comparison of the evaluated dimensions with their physical ones reveals that the results of the present theory are more realistic than those of the Lewin's theory.

The strict expressions of the phase constants obtained in this paper can be utilized for the design of practical waveguide twists.

Appendix
Derivation of (4)

By substituting (2) into (1), the integrands in each side of (1) can be expressed as

$$\begin{aligned}
\operatorname{rot} E\cdot\operatorname{rot} E^* &= \left|\frac{\partial\Phi_\eta^{(0)}}{\partial X}\right|^2 + \beta_T^2\left|\Phi_\eta^{(0)}\right|^2 \\
&+ \alpha^2\Bigg\{\left|\frac{\partial\Phi_\xi^{(1)}}{\partial Y}\right|^2 + \left|\frac{\partial\Phi_\eta^{(1)}}{\partial X}\right|^2 + \left|\frac{\partial\Phi_\xi^{(1)}}{\partial X}\right|^2 \\
&+ \left|\frac{\partial\Phi_\xi^{(1)}}{\partial Y}\right|^2 + X^2\left|\frac{\partial\Phi_\eta^{(0)}}{\partial X}\right|^2 + Y^2\left|\frac{\partial\Phi_\eta^{(0)}}{\partial X}\right|^2 \\
&+ \beta_T^2\left(\left|\Phi_\xi^{(1)}\right|^2 + \left|\Phi_\eta^{(1)}\right|^2\right) + 2\frac{\partial\Phi_\eta^{(1)}}{\partial X}\cdot\frac{\partial\Phi_\xi^{(1)}}{\partial Y} \\
&- 2Y\frac{\partial\Phi_\eta^{(0)}}{\partial X}\cdot\frac{\partial\Phi_\xi^{(1)}}{\partial Y} - 2X\frac{\partial\Phi_\eta^{(0)}}{\partial X}\cdot\frac{\partial\Phi_\xi^{(1)}}{\partial X} \\
&+ j2\beta_T\Bigg[\Phi_\eta^{(1)}\frac{\partial\Phi_\xi^{(1)}}{\partial Y} + \Phi_\xi^{(1)}\frac{\partial\Phi_\xi^{(1)}}{\partial X} + Y\Phi_\eta^{(0)} \\
&\cdot\left(\frac{\partial\Phi_\eta^{(1)}}{\partial X} - \frac{\partial\Phi_\xi^{(1)}}{\partial Y}\right) - Y\Phi_\eta^{(1)}\frac{\partial\Phi_\eta^{(0)}}{\partial X} - X\Phi_\xi^{(1)}\frac{\partial\Phi_\eta^{(0)}}{\partial X}\Bigg]\Bigg\} \\
&+ \alpha^4(X^2+Y^2)\left(\left|\frac{\partial\Phi_\xi^{(1)}}{\partial Y}\right|^2 + \left|\frac{\partial\Phi_\eta^{(1)}}{\partial X}\right|^2 + 2\frac{\partial\Phi_\xi^{(1)}}{\partial Y}\cdot\frac{\partial\Phi_\eta^{(1)}}{\partial X}\right)
\end{aligned}$$

$$\begin{aligned}
E\cdot E^* &= \left|\Phi_\eta^{(0)}\right|^2 \\
&+ \alpha^2\Big(\left|\Phi_\xi^{(1)}\right|^2 + \left|\Phi_\eta^{(1)}\right|^2 + \left|\Phi_\xi^{(1)}\right|^2 \\
&+ X^2\left|\Phi_\eta^{(0)}\right|^2 - 2X\Phi_\eta^{(0)}\Phi_\xi^{(1)}\Big) \\
&+ \alpha^4\Big(Y^2\left|\Phi_\xi^{(1)}\right|^2 + X^2\left|\Phi_\eta^{(1)}\right|^2 + 2XY\Phi_\xi^{(1)}\Phi_\eta^{(1)}\Big) \qquad \text{(A1)}
\end{aligned}$$

in which the terms of α and α^3 are found to vanish.

The scalar functions of the form $\Phi^{(0)}$ and $\Phi^{(1)}$ have been found [2] to be

$$\Phi_\eta^{(0)} = \sin\theta$$

$$\Phi_\xi^{(1)} = j\frac{\beta_g a^2}{\pi^2}\left\{\left(\theta-\frac{\pi}{2}\right)\cos\theta - \sin\theta + \frac{4}{\pi}\sum_{m=0,2,\cdots}^{\infty}\frac{\epsilon_m\cos m\theta}{(m^2-1)^2}\right.$$

$$\left.\cdot\frac{\cosh\left[\phi\sqrt{m^2-1}\left(\frac{2}{\pi}\psi-1\right)\right]}{\cosh\left(\phi\sqrt{m^2-1}\right)}\right\}$$

$$\Phi_{\eta}^{(1)} = j\frac{\beta_g a^2}{\pi^2}\left\{\frac{b}{a}\left(\theta-\frac{\pi}{2}\right)\left(\psi-\frac{\pi}{2}\right)\sin\theta+\frac{16}{\pi}\sum_{m=2,4,\cdots}^{\infty}\cdot\frac{m\sin m\theta}{\left(\sqrt{m^2-1}\right)^5}\cdot\frac{\sinh\left[\phi\sqrt{m^2-1}\left(\frac{2}{\pi}\psi-1\right)\right]}{\cosh\left(\phi\sqrt{m^2-1}\right)}\right\}$$

$$\Phi_{\xi}^{(1)} = \frac{a}{\pi}\left\{\left(\theta-\frac{\pi}{2}\right)\sin\theta+\frac{8}{\pi}\sum_{m=2,4,\cdots}^{\infty}\frac{m\sin m\theta}{(m^2-1)^2}\cdot\frac{\cosh\left[\phi\sqrt{m^2-1}\left(\frac{2}{\pi}\psi-1\right)\right]}{\cosh\left(\phi\sqrt{m^2-1}\right)}\right\} \quad \text{(A2)}$$

where

$$\theta=\frac{\pi}{a}\left(X+\frac{a}{2}\right),\quad \psi=\frac{\pi}{b}\left(Y+\frac{b}{2}\right)$$

$$\phi=\frac{\pi b}{2a},\quad \epsilon_m=\begin{cases}1 & (\text{for } m=0)\\ 2 & (\text{for } m\neq 0)\end{cases}.$$

On substituting (A2) and (3) into (A1), each integrand in (A1) is expressed in terms of the variables X, Y, and the unknown constant A_2, together with the remaining constants. After integrating (1) and neglecting the terms of α^4, we get (4) of the text.

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